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Note

Nonuniform Mesh Systems*

In the paper by Crowder and Dalton [1] the conclusion is reached that a uniform square mesh is the best grid for the Poiseuille flow problem. If the truncation error is examined, utilizing the exact solution for Poiseuille flow to evaluate the derivatives, one finds that the error for this problem is a constant if a uniform mesh is used. If a variable mesh size is employed, the truncation error for this problem will vary with radial position. This is the reason for the optimality of the uniform mesh for this problem. The problem investigated by Crowder and Dalton is really the exceptional case and their conclusions about nonuniform meshes can, in general, be very misleading. For other problems, where a uniform mesh gives a truncation error which changes greatly in the region of solution (such as the problems of Thoman and Szewczyk [3]) the use of a nonuniform mesh is yet to be proved inferior.

Also, in the paper by Crowder and Dalton [1], a very crude nonuniform mesh system was employed. In order to keep the formal truncation error locally of order Δr_i^2, the following restriction must be met:

$$\Delta r_{i+1} = f \Delta r_i , \qquad f = (1 + \alpha \Delta r_i),$$

where α is of order one. This shows that the mesh size should change slowly. Crowder and Dalton doubled the mesh size from $\Delta r = 0.05$ to 0.1, giving $f = 2$. The above equation indicates that $f \approx 1.05$ would be required for an error of order $(\Delta r_i)^2$. (In fact, the Crowder and Dalton mesh change gives an error of order Δr_i.) If solutions to other problems are obtained with a variable grid which obeys the above restriction, it appears possible to reduce the number of points used in a uniform mesh and still obtain a solution with as much accuracy.

Finally, it is pointed out that a significant improvement in the accuracy of solutions can often be obtained by transforming the independent variables, rather than by changing the mesh. Both of these approaches have the same motivation of increasing the density of calculated points in regions of high curvatures, and the approaches are frequently mistaken for one another in the literature. They are fundamentally different, however. For the Poiseuille problem, consider introducing

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a new independent variable $R = r^2$, and a transformed vorticity $\bar{G} = rG$. Since the exact solution for stream function F becomes

$$F = \tfrac{1}{2}R - \tfrac{1}{4}R^2,$$

the usual second-order accurate spatial differencing in R will give the exact solution, regardless of ΔR. It is thus seen that the use of transformed independent coordinates can be more effective in improving the accuracy of numerical solutions than the use of a nonuniform mesh, since the formal truncation error is not degraded in the transformed coordinate system. Both of these approaches will affect the phase error, however, and it is probably true that the uniform grid is best for the stability problem studied by Crowder and Dalton [2] where an oscillatory disturbance is introduced.

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